

$$s_u[n] = \sum_{l=-\infty}^{\infty} A_u[l] h_u[n - lN] . \quad (101)$$

$N = M + P$  is the length of the time domain symbol plus the Guard Interval.  $h_u[n]$  is the pulse response of the sound  $u$  and may be written as

$$h_u[n] = \begin{cases} e^{j \frac{2\pi}{M} un} & \text{for } n = 0, 1, \dots, M-1 \\ 0 & \text{else.} \end{cases} \quad (102)$$

In order to guarantee a real value time signal,  $A_{u^*}[n]$  is applied to the sound  $M-u$ ,  $M$  being the block length of the IDFT processing.

$$s_u[n] = \sum_{l=-\infty}^{\infty} \left( A_u[l] h_u[n - lN] + A_{u^*}[l] h_{M-u}[n - lN] \right) \quad (103)$$

$$= \sum_{l=-\infty}^{\infty} \left( A_u[l] h_u[n - lN] + A_u^*[l] h_u^*[n - lN] \right) \quad (104)$$

In order to reduce the power density spectrum (PSD) of  $s_u[n]$  within the fade-out range, the sounds  $i, i \in K_I$  are used to transmit the compensation signals. The set  $K_I$  contains the index of those sounds that should be used for compensation.

$$\begin{aligned}
 s_u[n] = & \sum_{l=-\infty}^{\infty} \left( A_u[l] h_u[n - lN] \right. \\
 & + A_u[l] \left( [c_u[0]]_1 h_{i_1}[n - lN] + \cdots + [c_u[0]]_I h_{i_I}[n - lN] \right) \\
 & \vdots \\
 & + A_u[l - R + 1] \left( [c_u[R - 1]]_1 h_{i_1}[n - lN] + \cdots + [c_u[R - 1]]_I h_{i_I}[n - lN] \right) \Big) \\
 & + CC
 \end{aligned} \tag{105}$$

The first row corresponds to a conventional DMT signal when  $A_u[l]$  is applied to the sound  $u$ .  $I$  is the number of elements that are contained in the set  $K_I$ . In the second line, the actual data  $A_u[l]$  are not transmitted by the sound  $u$ , but weighted versions of  $A_u[l]$  are transmitted by the sounds  $i, i \in K_I$ . The weighting of  $A_u[l]$  is effected by the weighting vector  $c_u[0]$ .  $[c_u[0]]_i$  is the  $i^{\text{th}}$  coordinate of the vector  $c_u[0]$ . The transmission of the weighted versions of  $A_u[l]$  by the sounds  $i, i \in K_I$  should minimize the effect of  $A_u[l]$  within the fade-out range. The following lines correspond to the transmission of the weighted and delayed  $A_u[l-r]$ ,  $r = 1, 2, \dots, R-1$ . This should minimize the effect of the past values  $A_u[l-r]$  within the fade-out range. The number  $R$  determines the memory. The optimal selection of the weighting factors  $c_u[r]$ ,  $r = 0, 1, \dots, R-1$  will be explained herein after.

A more compact notation can be achieved by using vectors.

$$s_u[n] = \sum_{l=-\infty}^{\infty} \left( A_u[l] h_u[n - lN] + \sum_{r=0}^{R-1} A_u[l - r] c_u^T[r] h_I[n - lN] \right) + \text{CC} \quad (106)$$

The column vector  $h_I[n]$  contains the pulse responses at the instant of time  $n$  of the sounds used for compensation and may be written as

$$h_I^T = [h_{i_1}[n] \ h_{i_2}[n] \ \dots \ h_{i_I}[n]] \quad \text{with} \quad \{i_1, i_2, \dots, i_I\} = \mathcal{K}_I. \quad (107)$$

In the case considered up to now, only one sound  $u$  is being transmitted. Now we will discuss the case in which not only one sound, but all the sounds  $u$ ,  $u \in \mathcal{K}_u$  are transmitted.

$$s[n] = \sum_{k \in \mathcal{K}_u} s_u[n] \quad (108)$$

$$\begin{aligned} &= \sum_{k \in \mathcal{K}_u} \left( \sum_{l=-\infty}^{\infty} \left( A_u[l] h_u[n - lN] + \sum_{r=0}^{R-1} A_u[l - r] c_u^T[r] h_I[n - lN] \right) + \text{CC} \right) \\ &= \sum_{l=-\infty}^{\infty} \left( A^T[l] h_u[n - lN] + \sum_{r=0}^{R-1} A^T[l - r] C[r] h_I[n - lN] \right) + \text{CC} \end{aligned} \quad (109)$$